Non-Machian, Lorentz-invariant Inertia: The first step towards the theory of GravitoElectroMagnetism

Shahriar S. Afshar

IRIMS, P.O. Box 1281, Brookline, MA 02446
Telephone: (617) 783-4107; E-mail: afshar@post.harvard.edu

The centrally symmetrical momentum field of radiation around a point-like locally isotropic source undergoes an axisymmetric transformation due to relativistic Doppler shift when seen by a moving observer. In order to keep the source force-free, this transformation assigns to it a nonlinear velocity-dependent momentum vector. A novel concept, Radiation-Induced Mass (RIM), is introduced, and equivalence of the derived mass term with the relativistic $m = \gamma\, m_0$ is proved. In addition to the origin of inertia, the application of RIM in solving several other problems such as the structure of spacetime, the nature of "dark matter," the large-scale dynamics of galaxies, and the arrow of time, is discussed.

INTRODUCTION

The origin of inertia, the familiar tendency of matter to resist acceleration, remains as much a mystery today as it did in 1638 when Galileo first proposed it to explain the uniform rate of free-fall for all bodies. In 1687, in his second law of motion, Newton formulated this concept essentially as follows: if one applies an external force **F** to a body of matter content (mass) m, that body accelerates in such a manner that $\mathbf{a} = \frac{\mathbf{F}}{m}$. Newton postulated that mass is a fundamental property of matter and that space and time are both absolute. Under the latter assumption,

$$\mathbf{a} = \frac{d^2\mathbf{r}}{d\,t^2} \tag{1}$$

where **r** is the absolute position three-vector of the body with respect to an arbitrary point in absolute space, and t is the absolute time.

The failure of the classic Michelson-Morley interferometer experiment to find the physical properties of the so-called "luminiferous ether" (the 19th century version of the absolute frame of reference) led to the abandonment of the concept of absolute space in the early 1900s. This meant that the Galilean transformation used by Newton to derive Eq. (1) was incomplete. In 1905, Einstein used a spacetime paradigm to derive the Lorentz transformation and the correct laws of kinematics for bodies moving at a constant rectilinear velocity. The rest frames of these bodies are called Inertial Frames (IF's). It is obvious that for any IF, $\mathbf{F} = 0$. In light of the universal validity of the Lorentz transformation, it naturally follows that in the absence of absolute space only relative motions including accelerations have any physical meaning. But, in the case of inertial forces, one can not use the relative acceleration of an observer and an accelerating body to derive the force acting on that body, for if relative acceleration were the cause of inertia, then it would have been possible for an observer to induce inertial forces on objects, or to cause charged particles to radiate, simply by accelerating towards them. This is not the case in any physical system so far as we know. Ernst Mach (1872) proposed a thought-provoking solution to the problem of relativity of acceleration. In developing his General Theory of Relativity (GTR), Einstein (1918) adopted Mach's epistemological views for

CP458, *Space Technology and Applications International Forum– 1999*
edited by Mohamed S. El-Genk

acceleration term such that for an IF, $\mathbf{a} = d^2(\frac{\sum m\,\mathbf{r}}{\sum m})/dt^2 = 0$, where $\mathbf{r}$ is the relative position three-vector of the IF with respect to the other bodies in the universe, m is the corresponding mass of each body, and t is the pre-relativistic notion of time (Mach, 1942). The exhaustive details of the correct mathematical derivation of MP, and its role in GTR, is still a matter of intense debate (Barbour, 1995). A new derivation of inertia is introduced here that renders MP superfluous.

MP represents an attempt to explain the origin of inertia without utilizing the concept of absolute space, by means of relativizing inertia as a "kind of mutual interaction of bodies," as Einstein put it in a letter to Mach in 1913 (Holton, 1972). Under MP, the existence of inertia is solely dependent on the presence of other massive objects in the cosmos and not on any local mechanisms. Therefore, if inertia could be derived locally by means of parameters that are independent of the global structure of the universe, MP would essentially be violated. In this paper, I show that non-Machian inertia can be derived for any source of radiation, as a by-product of its own emission process, which is necessarily independent of the distribution of mass in the universe. Every source of radiation has an inertial mass, caused exclusively by its radiant flux, which behaves exactly according to the relativistic mass term $m = \gamma\, m_o$, where m_o is the rest mass of the source due to the radiation, $\gamma = \sqrt{\frac{1}{1-\beta^2}}$, and $\beta = \frac{v}{c}$. In other words, the dynamics of a source of radiation is most accurately defined by its own radiant flux and its radiation field as the frame of reference (internal influences), rather than any radiation absorbed from other sources of roughly the same radiant power (external influences). We may refer to the concept that the rest mass is a linear function of the local radiant flux as Radiation-Induced Mass (RIM). It is important to note that RIM does not exclude the possible influence of the other bodies on the inertia of a particle of matter, only that it disallows "mass-energy there, [to completely] rule inertia here" (Ciufolini *et al.*, 1995) RIM, in contrast with GTR, implies that any influence of the other bodies in the universe on the local inertia of another body can only be minimally reductive.

According to the conventional interpretation of MP, the inertia of a radiating source on the Earth must exhibit a direction-dependent anisotropy of the order of about 10^{-10} to 10^{-5} due to the higher concentration of masses at the center of our own galaxy (Hughes *et al.*, 1960). Investigations of Hughes-Drever type have shown that the inertial masses of bodies (which include sources of radiation) are isotropic to an upper limit precision of one part in 10^{25} (Drever, 1961). This implies that, in order to avoid anisotropy of the local inertia due to the global asymmetries, all of the inertia associated with a source of radiation must be attributed to non-Machian causes. Since sources of radiation are ubiquitous, it logically follows that inertia of all physical systems must also be non-Machian in origin.

RIM permits the derivation of a physically rigorous acceleration term, that neither involves the traditional interpretation of acceleration used in Eq. (1) nor the large-scale distribution of matter on which Mach's arguments depend. Consequently, RIM predicts that the only physically relevant acceleration is that of a source with respect to its own field of radiation. The radiation field of a source is itself a Non-Machian, Lorentz-invariant (NML) frame of reference that satisfies all the requirements of Special Relativity and avoids the complications of MP. This hitherto uninvestigated frame of reference has the characteristics of a quantized future-pointing null cone in the Minkowski four-space, that connects the source to the other bodies in the universe in a unidirectional and causal sense.

One can further advance the argument, and generalize by allowing RIM to completely explain the inertial rest masses of material particles. To do this we must treat a particle of matter as a point-like source of radiation at which the emitted energy is being continuously created. Where, according to Einstein's geometrodynamics we had three physical realities namely energy, matter and spacetime, now we can assume only one physical reality; for it can be shown, using RIM, that matter and spacetime are one and the same, simply energy (more specifically electromagnetic energy). Remarkably, the dominance of the exitance (radiant flux density of the radiation leaving the source) compared with irradiance (radiant flux density absorbed by the source from the ambient radiation field of the other nearby sources) introduces a natural asymmetry between the past and the future events that essentially leads to a real and physically rigorous arrow of time, which has thus far proved illusive (Price, 1997). Furthermore, the implications of treating material particles as sources of radiation has significant cosmological consequences. The nature of "dark matter" and large-scale dynamics of galaxies in the universe are discussed in depth in a separate article (Afshar, 1999)

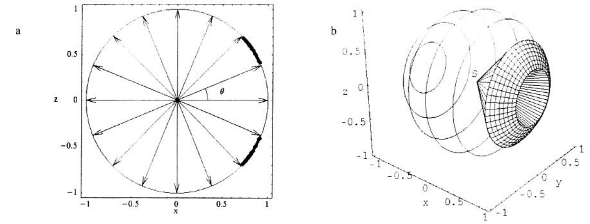

FIGURE 1(a). A cross-section of the sphere of radiation in *K*, showing the sectioning of the great hemicircle into $n = 8$ for easier visualization. The thickened arcs show the cross section of the shell surface A_2. Arrows depict the position three-vectors of the emitted photons in classical trajectories. (b) 3D rendition of the sphere of radiation in *K*, and the shell surface A_2. Note that photon flux N_i through A_i remain unchanged for all values of *i*.

DERIVATION OF THE NML MASS TERM

Imagine that *S*, a point-like source of coherent isotropic monochromatic electromagnetic radiation of wavelength λ_o and constant radiant flux Φ_S, is positioned at the origin of its inertial rest frame K. From hereon we shall only use three-vectors unless otherwise stated. We know from classical electromagnetism and quantum mechanics that each photon leaving the source carries with it a linear momentum of magnitude $|\mathbf{p}_o| = p_o = \frac{E_o}{c} = \frac{h}{\lambda_o}$ (when measured inside K), where E_o is the energy of the photon, c is the speed of light, and h is the Planck's constant. Now, envision an observer *O*, at rest in her inertial frame K', approaching S with a speed $|-\mathbf{v}| = v = \beta\, c$, where **v** is parallel to the mutual x and x' axes of K and K', respectively. For simplicity of argument, we exclude any motion along the other two spatial dimensions and allow the other two axes to be parallel. Assume further that the photon flux density of S measured in K is $\varphi = \frac{\Phi_S}{4\pi\, R^2 E_o}$, where *R* is an arbitrary radius of a sphere of radiation, the center of which is occupied by S, and $E_o = \frac{h\,c}{\lambda_o}$ is the energy of an individual photon. For the ease of our calculations, we divide the surface of this sphere of radiation into infinitesimally thin concentric spherical shells, each of which subtends an angle $\Delta\theta = \frac{\pi}{n}$ on the great circle of the sphere, where n is an arbitrary number of divisions (Fig. 1A). We arrange these shells in a fashion that the angle of each leading edge of the ith shell with respect to the velocity vector **v** is $\theta = i\,\Delta\theta$ in a counter-clockwise sense (Fig. 1B). It is clear that $i = 1,2,3,\ldots,n$, and that the entire sphere is covered by n concentric shells. The surface area of the ith shell is given by:

$$A_i = 2\pi\, R^2 \int_{(i-1)\Delta\theta}^{i\Delta\theta} d\theta\, \sin\theta\,, \tag{2}$$

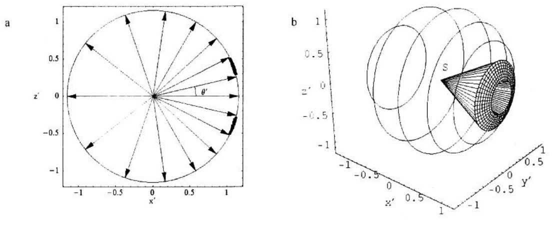

FIGURE. 2 (a). The cross-section of the same sphere of radiation as seen in K'. Again $n = 8$, and here the transformations are based on $\beta = 0.5$. (b) 3D rendition of the sphere of radiation in K'. and the shell surface A_2'. Note that in this case photon flux N_i' through A_i' decrease for increasing values of i.

so that the photon flux through A_i would simply be:

$$N_i = \varphi\ A_i = \Phi_S \frac{\lambda_0}{2h\ c} \int_{(i-1)\Delta\theta}^{i\Delta\theta} d\theta\ \sin\theta \cdot \tag{3}$$

Here, since the radiation is coherent, photons leave the source in coherent wave-fronts, such that in between each surge of photons, there is no emission. The shortest period of time between each surge is the period of the wave T_0, and therefore at any instant of emission the source emits $U_i = N_i\ T_0 = \dfrac{\lambda_0 N_i}{c}$ number of photons onto A_i, U_i being the photon dose of the ith shell. Hence:

$$U_i = \Phi_S \frac{\lambda^2}{2h\ c^2} \int_{(i-1)\Delta\theta}^{i\Delta\theta} d\theta\ \sin\theta \tag{4}$$

Now if we switch to IF of the observer O, we find that the locally symmetric radiation (or momentum) field of S has undergone an axisymmetric transformation according to the following three rules:

(i) Because of the so-called relativistic "starlight aberration," photons leaving S at an angle θ relative to the velocity vector **v** as measured in K are seen to leave S at an angle θ' when observed in K' (compare Fig. 1A and 2A), such that:

$$\cos\theta' = \frac{\beta + \cos\theta}{1 + \beta\cos\theta} \cdot \tag{5}$$

(ii) The sphere of radiation emitted at $t = t' = 0$, when spatial origins of K and K' coincide, evolves during time Δt. Any point on the surface of this sphere satisfies $\Delta R^2 = c^2\Delta t^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$ in K, and is observed to have the form $\Delta R'^2 = c^2\Delta t'^2 = \Delta x'^2 + \Delta y'^2 + \Delta z'^2$ when measured in K'. Of course, $\Delta R' = \gamma \, \Delta R$. Since we are only concerned with the angular distribution of the emitted photons, we allow $\Delta t \to 0$ (Fig. 2B).

(iii) The magnitude of the momenta of photons leaving S are measured by O to behave according to the following relationship:

$$|\mathbf{p}'| = p' = p_0 D\ (\theta', \beta), \tag{6}$$

where

$$D\ (\theta', \beta) = \frac{\sqrt{1-\beta^2}}{1-\beta\cos\theta'} \tag{7}$$

is the familiar relativistic Doppler shift function.

Since the radiation is assumed to be locally isotropic, the sum of momenta of photons leaving S measured inside K is $\mathbf{P} = \sum \mathbf{p}_0 = 0$, and thus the force acting on S is $\mathbf{F} = -\frac{d\mathbf{P}}{dt} = 0$. However, the same can not be said about $\mathbf{P}' = \sum \mathbf{p}'$ due to the effect of the above three transformations. In order to calculate $\mathbf{P}'$ we must take advantage of the fact that the photon dose through A_i' is the same as that through A_i, and therefore $U_i' = U_i$ (compare Fig. 1B and 2B). Also, we approximate momenta of photons through the ith shell surface to be represented by the photons passing through the middle of the shell, i.e.: for photons passing through A_i subtending $(i-1)\Delta\theta$ to $\Delta\theta$, we assume $\theta = (i - \frac{1}{2})\Delta\theta$ in Eq. (5). The accuracy of this approximation improves by an increase in n. In my calculations, I have used $n = 10^4$, although much smaller values produce satisfactory results. Because of the axisymmetric nature of the above transformations (x' being the axis of symmetry), we are only interested in $\mathbf{p}'_{x'} = \mathbf{p}'\cos\theta'$, since the other components of the linear momenta of the emitted photons cancel each other out. Utilizing Eq.s (2-7), we have:

$$\mathbf{P}' = \sum_{i=0}^{n} U_i\ \mathbf{p}_0 D\ (\theta', \beta)\cos\theta' \cdot \tag{8}$$

Now let us consider what the observer in K' would see when looking at S. Eq.(8) tells us that as far as O is concerned there is a force $\mathbf{F}' = -\frac{d\mathbf{P}'}{dt'} \neq 0$ acting on S. This is clearly an impossibility because since K and K' are both IF's at rectilinear motion along the mutual x and x' axes, we must have $\mathbf{F}' = \mathbf{F} = 0$. It follows therefore, that $\mathbf{P}'$ must be interpreted as the instantaneous linear momentum of the source itself.

Finally, in order to derive the mass term, we need to calculate $m' = \frac{\mathbf{P}'}{\mathbf{v}}$. Thus using Eq.s (4-8) we have:

$$m' = \Phi_S \frac{\lambda_0}{c^3}\Gamma, \tag{9}$$

where

$$\Gamma = \frac{\sqrt{1-\beta^2}}{2\beta}\sum_{i=0}^{n}\frac{\cos\theta'}{1-\beta\cos\theta'}\int_{(i-1)\Delta\theta}^{i\Delta\theta} d\theta\ \sin\theta \cdot \tag{10}$$

Strikingly, $\lim_{n\to\infty}\Gamma \equiv \gamma$. Therefore, Eq. 9 can be rewritten as $m' = \gamma\ m_0$ where

$$m_0 = \Phi_S \frac{\lambda_0}{c^3} \cdot \tag{11}$$

Eq. (11) demonstrates that the NML inertial rest mass derived by means of RIM behaves relativistically and is therefore locally identical in nature to the Special Relativistic Lorentz-invariant rest mass. Obviously, the measurement of Φ_S and λ_0 is an empirical challenge, for in reality, different species of elementary particles could well be composed of numerous sources of radiation, with different frequencies and polarization states. To recap, we can now define the NML inertia as the radiation-induced resistance **F**, of a source of radiation to an acceleration relative to its own field of radiation, the acceleration being $\mathbf{a} = \dfrac{\mathbf{F}\, c^3}{\Phi_S\, \lambda_0}$.

CONCLUSION

Apart from a major paradigm shift in the definition of matter, this approach provides a true and solid basis for the long-awaited unified theory of Gravity and Electromagnetism, which I have dubbed "GravitoElectroMagnetism" or (G.E.M.). This is possible due to the fact that upon a closer inspection, the equation for the local inertial rest mass Eq. (11) shows that although as far as the particle itself is concerned the local rest mass is determined by Φ and λ; however, its gravitational influence upon a body at distance r, is a radially-dependent function of its rest mass. The following example can clarify this point further:

Within a spherical volume of radius r centered on particle 1, in addition to its rest mass we have a radially-dependent mass component $m_1(r)$ associated with the energy field itself. It is easy to show that $m_1(r) = \Phi_1 \frac{r}{c^3}$. I suggest, that It is this peculiar component of the gravitational mass of any body of matter that plays the role of the so-called "dark matter" in the universe. Therefore, for particle 2 situated somewhere on the surface of the sphere, the gravitational mass of particle 1 is given by:

$$m_1 = \frac{\Phi_1}{c^3}(\lambda_0 + r) = m_0(1 + kr), \;\; k = \lambda_0^{-1}. \tag{12}$$

Thus it follows, that Newton's universal law of gravitation should be corrected to read:

$$F = G\frac{M_1\, M_2(1 + kr)}{r^2}, \tag{13}$$

where M_1 and M_2 are the rest masses of bodies 1 and 2 respectively, and G is the universal gravitation constant. Note that at relatively small distances (when $r << \lambda_0$, e.g. Solar neighborhood) Eq. (13) reduces to Newtonian limits so that $F \approx G\frac{M_1\, M_2}{r^2}$. At large distances however, Eq. (13) shows a striking deviation from the Newtonian law of gravity such that $F \approx G\frac{k\, M_1\, M_2}{r}$, implying a mean orbital velocity that remains constant at large distances. The rotation curves (spectrographic analysis of the rotating arms) of the Spiral galaxies, as well as the large-scale dynamics in the universe, suggest just such gravitational interactions and provide an approximate value for $k \approx 1.24 \pm 0.5\,\text{kpc}^{-1}$. This issue is exhaustively studied in a separate paper by the present author and shall be presented at the Relativistic Astrophysics Symposium to be held in Paris, France (Afshar, 1999). There are many more noteworthy problems that G.E.M. successfully solves which due to the page limitations of this publication cannot be related here and will be covered in a comprehensive paper to be submitted to PRA.

ACKNOWLEDGEMENTS

I wish to express my appreciation to the Faculty of Arts and Sciences Computer Services, Wolbach library, and Physics Research library of Harvard Graduate School of Arts and Sciences for the generous permission to use the facilities for this project. Partial funding was provided by D. W. Glazer, G. B. Davis, Anna and Richard Southgate, Jeremy Grantham, Christopher Darnell and Howard Fuguet to all of whom I am deeply obliged.

REFERENCES

Afshar, S. S., "Non-Machian, Lorentz-invariant Inertia: The Dark Matter connection," in *19th Texas Symposium on Relativistic Astrophysics* to be published in 1999.

Barbour, J. B., *Mach's Principle: From Newton's Bucket to Quantum Gravity*, edited by J. B. Barbour and H. Pfister, Boston, Birkhäuser, 1995, pp. 6-10.

Boyer, T. H., *Phys. Rev. D* **11**, 790-812 (1975).

Ciufolini, I., and Wheeler, J. A. , *Gravitation and Inertia,* Princeton, NJ, Princeton Univ. Press, 1995, pp. 250-251.

Drever, R. W. P., *Philosophical Magazine* **6**, 683-687 (1961).

Einstein, A., *Annalen der Physik* **55**, 240-253 (1918).

Holton, G., in *The twentieth century sciences: studies in the biography of ideas*, New York, Norton 1972.

Hughes, V.W., Robinson, H. G., and Beltran-Lopez, V., *Phys. Rev. Lett.* **4**, 342-346 (1960).

Mach, E., *Die Geschichte und die Wurzel des Satzes von der Erhaltung der Arbeit*, Prag, Calve'sche Buchhandlung, 1872, pp.160-183.

Mach, E., *The Science of Mechanics*, LaSalle, Ill., Open Court, 1942, pp.120-125.

Price, H., *Time's arrow and Archimedes' point,* New York, Oxford Univ. Press, 1997, pp. 3-21.